BASUDEV GODABARI DEGREE COLLEGE , KESAIBAHAL

Department of Computer Science

"SELF STUDY MODULE"

Module Details :

- Class – 5TH Semester
- Subject Name : COMPUTER SCIENCE
- Paper Name : DSE-I

UNIT – 2 : STRUCTURE

2.1 Bisection method,

2.2 Secant method

2.3 Regula–Falsi method

2.4 Newton–Raphson method

You Can use the Following Learning Video link related to above topic :

https://youtu.be/3jOc_FhOt5U

https://youtu.be/Eud16189QRA

https://youtu.be/FliKUWUVrEI

https://youtu.be/7eHuQXMCOvA

Look related link for better understanding

You Can also use the following Books :

1. S.S. Sastry, "Introductory Methods of Numerical Analysis", EEE , 5/ed.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, 6/e (2012)

And also you can download any book in free by using the following website.

- https://www.pdfdrive.com/

Unit-2

Bisection Method Definition

The bisection method is used to find the roots of a polynomial equation. It separates the interval and subdivides the interval in which the root of the equation lies. The principle behind this method is the intermediate theorem for continuous functions. It works by narrowing the gap between the positive and negative intervals until it closes in on the correct answer. This method narrows the gap by taking the average of the positive and negative intervals. It is a simple method and it is relatively slow. The bisection method is also known as interval halving method, root-finding method, binary search method or dichotomy method.

Let us consider a continuous function "f" which is defined on the closed interval [a, b], is given with f(a) and f(b) of different signs. Then by intermediate theorem, there exists a point x belong to (a, b) for which f(x) = 0.

Bisection Method Algorithm

Follow the below procedure to get the solution for the continuous function:

For any continuous function f(x),

- Find two points, say a and b such that $a < b$ and $f(a)* f(b) < 0$
- Find the midpoint of a and b, say "t"
- t is the root of the given function if f(t) = 0; else follow the next step
- Divide the interval [a, b] – If $f(t)*f(a) <0$, there exist a root between t and a
  – else if $f(t) *f (b) < 0$, there exist a root between t and b
- Repeat above three steps until f(t) = 0.

The bisection method is an approximation method to find the roots of the given equation by repeatedly dividing the interval. This method will divide the interval until the resulting interval is found, which is extremely small.

Bisection Method Example

Question: Determine the root of the given equation $x^2-3 = 0$ for $x \in [1, 2]$

Solution:

Given: $x^2-3 = 0$

Let $f(x) = x^2-3$

Now, find the value of f(x) at a= 1 and b=2.

$f(x=1) = 1^2-3 = 1 – 3 = -2 < 0$

$f(x=2) = 2^2-3 = 4 – 3 = 1 > 0$

The given function is continuous, and the root lies in the interval [1, 2].

Let "t" be the midpoint of the interval.

I.e., t = (1+2)/2

t =3 / 2

t = 1.5

Therefore, the value of the function at "t" is

f(t) = f(1.5) = $(1.5)^2$-3 = 2.25 – 3 = -0.75 < 0

If f(t)<0, assume a = t.

and

If f(t)>0, assume b = t.

f(t) is negative, so a is replaced with t = 1.5 for the next iterations.

The iterations for the given functions are:

Iterations	a	b	t	f(a)	f(b)	f(t)
1	1	2	1.5	-2	1	-0.75
2	1.5	2	1.75	-0.75	1	0.062
3	1.5	1.75	1.625	-0.75	0.0625	-0.359
4	1.625	1.75	1.6875	-0.3594	0.0625	-0.1523
5	1.6875	1.75	1.7188	-01523	0.0625	-0.0457
6	1.7188	1.75	1.7344	-0.0457	0.0625	0.0081
7	1.7188	1.7344	1.7266	-0.0457	0.0081	-0.0189

So, at the seventh iteration, we get the final interval [1.7266, 1.7344]

Hence, 1.7344 is the approximated solution.

Secant Method of Numerical analysis

- Last Updated : 18 Oct, 2021

Secant method is also a recursive method for finding the root for the polynomials by successive approximation. It's similar to the **Regular-falsi** method but here we don't need to check **$f(x_1)f(x_2)<0$** again and again after every approximation. In this method, the neighbourhoods roots are approximated by secant line or chord to the function **f(x)**. It's also advantageous of this method that we don't need to differentiate the given function **f(x)**, as we do in **Newton-raphson** method.

- Since convergence is not guaranteed, therefore we should put limit on maximum number of iterations while implementing this method on computer.

Example-1 :

Compute the root of the equation $x^2e^{-x/2} = 1$ in the interval [0, 2] using the secant method. The root should be correct to three decimal places.

Solution –

$x_0 = 1.42$, $x_1 = 1.43$, $f(x_0) = -0.0086$, $f(x_1) = 0.00034$.

Apply, **secant method,** The first approximation is,

$x_2 = x_1 - [(x_0 - x_1) / (f(x_0) - f(x_1)]f(x_1)$

$= 1.43 - [(1.42 - 1.43) / (0.00034 - (-0.0086))](0.00034)$

$= 1.4296$

$f(x_2) = -0.000011$ (–ve)

The second approximation is,

$x_3 = x_2 - [(x_1 - x_2) / (f(x_1) - f(x_2))]f(x_2)$

$= 1.4296 - [(1.42 - 1.4296) / (0.00034 - (-0.000011](-0.000011)$

$= 1.4292$

Since, $\mathbf{x_2}$ and $\mathbf{x_3}$ matching up to **three decimal places,** the required root is **1.429**.

Example-2 :

A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the secant method.

Solution –

We have, $x_0 = 0$, $x_1 = 1$, $f(x_0) = 1$, $f(x_1) = -3$

$x_2 = x_1 - [(x_0 - x_1) / (f(x_0) - f(x_1))]f(x_1)$

$= 1 - [(0 - 1) / ((1-(-3))](-3)$

$= 0.25$.

$f(x_2) = -0.234375$

The second approximation is,

$x_3 = x_2 - [(x_1 - x_2) / (f(x_1) - f(x_2))]f(x_2)$

$=(-0.234375) - [(1 - 0.25)/(-3 - (-0.234375))](-0.234375)$

$= 0.186441$

$f(x_3)$

The third approximation is,

x4 = x3 – [(x2 – x3) / (f(x2) – f(x3))]f(x3)

= 0.186441 – [(0.25 – 0.186441) / (– 0.234375) – **(0.074276)](– 0.234375)**

= 0.201736.

f(x4) = – 0.000470

The fourth approximation is,

x5 = x4 – [(x3 – x4) / (f(x3) – f(x4))]f(x4)

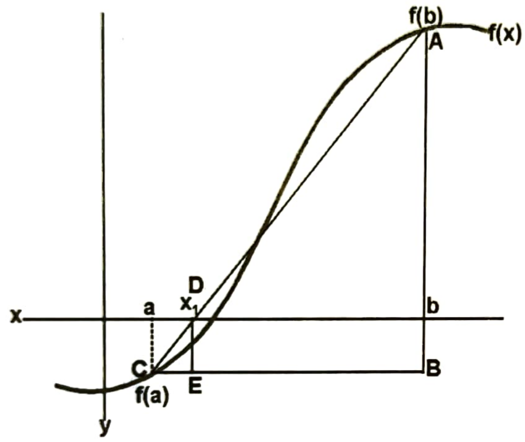
= 0.201736 – [(0.186441 – 0.201736) / (0.074276 – (– 0.000470)](– 0.000470)

= 0.201640

The **Regula–Falsi Method** is a numerical method for estimating the roots of a polynomial **f(x)**. A value **x** replaces the midpoint in the Bisection Method and serves as the new approximation of a root of **f(x)**. The objective is to make convergence faster. Assume that **f(x)** is continuous.

Algorithm for the Regula–Falsi Method: Given a continuous function **f(x)**

1. Find points **a** and **b** such that **a < b** and **f(a) * f(b) < 0**.
2. Take the interval **[a, b]** and determine the next value of x_1.
3. If $f(x_1) = 0$ then x_1 is an exact root, else if $f(x_1) * f(b) < 0$ then let $a = x_1$, else if $f(a) * f(x_1) < 0$ then let $b = x_1$.
4. Repeat steps 2 & 3 until $f(x_i) = 0$ or $|f(x_i)| \le DOA$, where **DOA** stands for **degree of accuracy**.



Observe that

$$EC / BC = E / AB$$

$$[\, x - a \,] / [\, b - a \,] = [\, f(x) - f(a) \,] / [\, f(b) - f(a) \,]$$

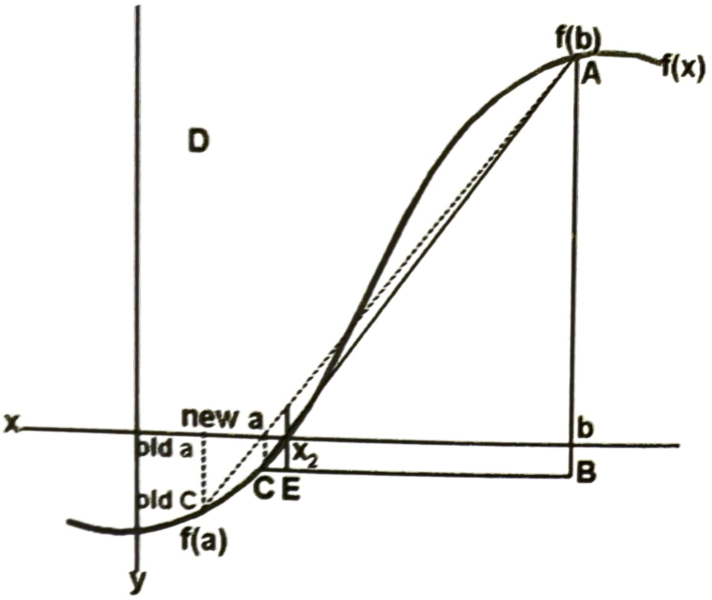
$$x - a = [\, b - a \,][\, 0 - f(a) \,] / [\, f(b) - f(a) \,]$$

$$x = a + [\, b - a \,][\, - f(a) \,] / [\, f(b) - f(a) \,]$$

$$x = a - [\, b - a \,]\, f(a) / [\, f(b) - f(a) \,]$$

Note that the line segment drawn from **f(a)** to **f(b)** is called the **interpolation line**.

Graphically, if the root is in $[\, a, x_i \,]$, then the next interpolation line is drawn between $(\, a, f(a) \,)$ and $(\, x_i, f(x_i) \,)$; otherwise, if the root is in $[\, x_i, b \,]$, then the next interpolation line is drawn between $(\, x_i, f(x_i) \,)$ and $(b, f(b))$.



EXAMPLE: Consider **f(x) = x^3 + 3x – 5**, where **[a = 1, b = 2]** and **DOA = 0.001**.

i	a	x	b	f(a)	f(x)	f(b)
1	1	1.1	2	– 1	– 0.369	9
2	1.1	1.13544668587896	2	– 0.369	– 0.129797592130931	9
3	1.13544668587896	1.14773797024856	2	– 0.129797592130931	– 0.0448680509813286	9
4	1.14773797024856	1.15196570867269	2	– 0.0448680509813286	– 0.0154155863909917	9
5	1.15196570867269	1.15341577448	2	– 0.0154155863909917	– 0.0052852985292482	9
6	1.15341577448	1.15391264384212	2	– 0.0052852985292482	– 0.00181077883487646	9
7	1.15391264384212	1.15408284038531	2	– 0.00181077883487646	– 0.000620231485743084	9